PREDVODITELEV HYDRODYNAMICS AT HIGH VELOCITIES

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The Predvoditelev hydrodynamics is extended to the steady-state flow of an incompressible fluid at relativistic velocities.

The behavior of a system of N interacting particles is described in statistical physics by the Liouville equation. A successive conversion from the Liouville equation to the Boltzmann equation and then to the equations of hydrodynamics represents a "short-cut description" [1] with a loss of some information about the system behavior. In [2] Predvoditelev has pointed out one such special but important case where information is lost during the conversion from the Boltzmann equation to the equations of hydrodynamics, namely the case where the gradients of the molecule transport velocities vary appreciably along the free path or over the extent of a characteristic volume element. This situation prevails during the flow of a rarefied gas, during a flow with vortex generation, or during a flow at sufficiently high velocities. The first two cases are covered by the Predvoditelev universal equations of hydrodynamics [2]. The third case, motion at high velocities, involves relativistic hydrodynamics. In this article here we extend Predvoditelev's ideas to the equations of steady-state relativistic hydrodynamics for an incompressible fluid.

Nonrelativistic Predvoditelev Hydrodynamics. In [2] Predvoditelev has used the Maxwell method for extending the equations of hydrodynamics to the case where the gradients of transport velocities within a hydrodynamic volume element are high. For an ideal compressible fluid these equations are

$$\frac{\partial v_\alpha}{\partial t} + (1-\beta)\, v_\gamma \frac{\partial v_\alpha}{\partial x^\gamma} - \beta v_\alpha \frac{\partial v_\gamma}{\partial x^\gamma} = -\frac{1}{\rho}\cdot\frac{\partial p}{\partial x^\alpha}, \tag{1}$$

and the continuity equation remains unchanged

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x^\gamma}(\rho v_\gamma) = 0. \tag{2}$$

The parameter of nonideal continuum β in Eq. (1)

$$|\beta| = \frac{3}{2}\,\mathrm{Kn}\,\mathrm{M}, \tag{3}$$

where Kn is the Knudsen number and M is the Mach number.

When $\beta = 0$ in (1), then

$$\frac{\partial v_\alpha}{\partial t} + v_\gamma \frac{\partial v_\alpha}{\partial x^\gamma} = -\frac{1}{\rho}\cdot\frac{\partial p}{\partial x^\alpha} \tag{4}$$

and we have the ordinary Euler equations; when $\beta = 2$, then

$$\frac{\partial v_\alpha}{\partial t} - v_\gamma \frac{\partial v_\alpha}{\partial x^\gamma} - 2v_\alpha \frac{\partial v_\gamma}{\partial x^\gamma} = -\frac{1}{\rho}\,\frac{\partial p}{\partial x^\alpha} \tag{5}$$

and we have the Kesterin hydrodynamics.

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If we now consider an incompressible fluid, then (2) becomes

$$\frac{\partial v_\gamma}{\partial x^\gamma} = 0, \tag{6}$$

and Eqs. (1) become

$$\frac{\partial v_\alpha}{\partial t} + (1-\beta)\, v_\gamma \frac{\partial v_\alpha}{\partial x^\gamma} = -\frac{1}{\rho} \cdot \frac{\partial p}{\partial x^\alpha}. \tag{7}$$

To this must be added the equation of state

$$F(p,\ \rho,\ T) = 0. \tag{8}$$

Universal Equations of Steady-State Relativistic Hydrodynamics. It has become feasible to generalize, in the Predvoditelev sense, the equations of steady-state relativistic hydrodynamics for an incompressible fluid owing, essentially, to the discovery by Frankl' [3] and further study by Shikin [4] of the possibility to reduce the equations of steady-state relativistic hydrodynamics for some effective fluid. The generalization shown here represents the solution to the reverse problem.

We introduce, as in [4], the following quantities:

$$\rho = \frac{m^2c^2n^2}{w}, \tag{9}$$

$$v_\alpha = \frac{wu_\alpha}{mcn}, \tag{10}$$

where m is the standstill mass of particle; n and w are, respectively, the number of particles and the relativistic thermal function of the specific volume; u_α are the space components of four-component velocities; and c is the velocity of light.

Inserting (9) and (10) into (6) and (7) yields the equation of steady-state continuity

$$\frac{\partial (nu^\alpha)}{\partial x^\alpha} = 0 \tag{11}$$

and the equations of steady-state hydrodynamics

$$(1-\beta)\, nu^\gamma \frac{\partial}{\partial x^\gamma}\left(\frac{w}{n} u^\alpha\right) = -\frac{\partial p}{\partial x^\alpha}. \tag{12}$$

It has been assumed in the derivation of (11) and (12) that

$$n = \text{const}, \qquad \frac{p}{nmc^2} = \text{const}. \tag{13}$$

Equations (11) and (12) are the universal Predvoditelev equations of steady-state relativistic hydrodynamics for an incompressible fluid. When $\beta = 0$, then (12) and (11) become, respectively, the ordinary equations of steady-state relativistic hydrodynamics and the equation of steady-state relativistic continuity [5-6]. This generalization of the equations of steady-state relativistic hydrodynamics will also apply to weakly compressible fluid: where the term $\beta v_\alpha (\partial v_\gamma / \partial x_\gamma)$ is small as compared to all other terms in these equations.

NOTATION

- v_α is the three-component velocity;
- u_α is the four-component velocity;
- p is the pressure;
- ρ is the density;
- β is the Predvoditelev number (constant hydrodynamic parameter).

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